An effective scalar magnetic interaction for resonantly trapped atoms

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(Dated: March 26, 2010.)

Atoms can be trapped using a combination of static and rotating magnetic fields. A theoretical analysis is performed of a rotating polarisation axis which is used to eliminate regions of zero coupling. A similar result is found using linear polarisation, but in the case of circular polarisation no orientational dependence in the coupling remains when on resonance.

PACS numbers:

I. INTRODUCTION

Trapping atoms by dressing them with rf radiation has been realised with a wide range of experimental and theoretical configurations [1–21]. The first experimental trapping with rf dressing [3] resulted in an egg-shell type potential with the atoms confined to the surface of the shell. Gravity caused the atoms to occupy the lower part of the egg-shell. Tubes of dressed atoms have been used to show interference effects [4, 11, 19], and there have been several designs and realisations of dressed atom ring-traps [7, 10, 13, 15].

The wide range of trapping topologies and shapes emerging is due to complex vector relationship between the oscillating and static magnetic fields used. These fields can change both in magnitude and in direction over a region of space. For the case of linearly polarised rf with a field $\mathbf{B_{rf}}(\mathbf{r})\cos\omega_{rf}t$, the interaction strength in the rotating wave approximation (RWA) is governed by a Rabi frequency Ω which may be written as

$$\Omega(\mathbf{r}) = \frac{|g_F|\,\mu_B}{2\hbar}B_{rf}(\mathbf{r})\sin\theta(\mathbf{r}) \equiv \Omega_0(\mathbf{r})\sin\theta(\mathbf{r})\,, \quad (1)$$

where $\theta(\mathbf{r})$ is the angle between the static field $\mathbf{B_0}(\mathbf{r})$ and the oscillating magnetic field. In equation (1) g_F is the usual Landé g-factor and μ_B is the Bohr magneton and clearly the maximum possible Rabi frequency at a location $\mathbf{r}$ is

$$\Omega_0(\mathbf{r}) = \frac{|g_F|\,\mu_B}{2\hbar}B_{rf}(\mathbf{r})\,. \quad (2)$$

The condition $\Omega_0(\mathbf{r}) \ll \omega_{rf}$ is required for the RWA to be valid [22]. The Rabi frequency $\Omega(\mathbf{r})$ shows clear spatial and orientational dependence. In this case the uppermost dressed state potential is simply given by

$$\mathcal{E}(\mathbf{r}) = \hbar F\left[(\omega_{rf} - \omega_0(\mathbf{r}))^2 + \Omega^2(\mathbf{r})\right]^{1/2}\,, \quad (3)$$

where $\omega_0(\mathbf{r})$ characterises the energy of the atom in the static field through

$$\omega_0(\mathbf{r}) = \frac{g_F\mu_B B_0(\mathbf{r})}{\hbar}\,. \quad (4)$$

The factor $\omega_{rf} - \omega_0(\mathbf{r})$ in equation (3) represents a spatially varying detuning. With this detuning the RWA is valid if $\Omega_0(\mathbf{r}), |\omega_{rf} - \omega_0(\mathbf{r})| \ll \omega_{rf}$. From equation (3) we see that, in a region of fairly uniform Rabi frequency, the defining feature of the trap is the region of resonance, i.e. an iso-B surface where $\omega_0(\mathbf{r}) = \omega_{rf}$. This picture gets modified in a complex way when the Rabi frequency is spatially varying, too.

In this paper we will pay particular attention to the situation which can arise when the static field $B_0(\mathbf{r})$ has the same orientation as the oscillating magnetic field $B_{rf}(\mathbf{r})$. In that case equation (1) shows that the Rabi frequency is zero, since the angle θ is zero. As a consequence the dressing breaks down and an atom is no longer trapped by dressed potentials. This would usually happen in a small region of space where the two magnetic fields align. (We note that the relatively "small" size of this region of atom loss is due to the exponential dependence of Landau-Zener losses on coupling squared [12].) Such a location has sometimes been termed a *hole* in the dressed potentials and it is usually undesirable, though in Ref. [12] two such holes were used to facilitate evaporative cooling in the dressed rf trap. The holes were arranged to be at the sides of the trap so that only the most energetic atoms reached them under the influence of gravity. More generally, the holes tend to arise when the relative angle between the magnetic fields varies considerably in space.

We should emphasise that these holes are *not inevitable* in dressed rf potentials. They can be avoided by appropriate use of bias fields: e.g. near the centre of a Ioffe-Pritchard magnetic trap [2, 3]. And they can also be avoided if the trapping region is different from the resonant region: for example, if the rf frequency ω_{rf} is below resonance for the centre of the trap [4].

If we switch from linear to circularly polarised rf, there is still potential for holes to be present. In a simple quadrupole trap (with coils in an anti-Helmholtz configuration) the number of holes reduces from two to one [7]. The difference is that the Rabi frequency in equation (3) is now

$$\Omega(\mathbf{r}) = \Omega_0(\mathbf{r})\left(1 + \cos\theta(\mathbf{r})\right) \quad (5)$$

which replaces equation (1). In equation (5) $\Omega_0(\mathbf{r})$ is the maximum possible Rabi frequency in the linear case as already given in equation (2). Thus we note that $\Omega(\mathbf{r})$, as given by equation (5), has a maximum value which is twice that of the linear case (see equation (1)) for a rf field with the same peak amplitude $B_{\rm rf}$. This is because in the linear case we always have to make the RWA, and reject the counter-rotating term, while in the circular polarisation case the RWA is not needed for the maximum coupling (σ_+) orientation. In equation (5) the angle θ is now the angle between the circular polarisation axis $\hat{\mathbf{e}}_c$ (which is perpendicular to the plane of rotation of $B_{\rm rf}(\mathbf{r})$) and the static field $\mathbf{B}_0(\mathbf{r})$. This means that when the rf rotates clockwise about the static magnetic field $\mathbf{B}_0(\mathbf{r})$ the maximum coupling (2) is achieved (for positive g_F); then as the plane of rotation is itself turned away, the coupling smoothly reduces to zero, a value reached when the axis rotation has completely turned around.

Thus, for dressed rf traps formed in regions where the static field varies over all directions, the holes cannot be removed by simply changing rf polarisation. For large traps the holes can be moved to places where the atoms will not reach them because of gravity. However, a procedure to remove holes of this kind could be important when dressing very small magnetic traps [23]. In that case gravity cannot be relied on to keep atoms away from the holes. An example arises when we consider dressed traps made with fields from magnetic nano-wires [24]. In this case the field gradients are so strong that for normal rf frequencies only very small (micron-scale) traps would be made by dressing: atoms at micro-Kelvin temperatures would then explore all parts of the dressed potential, find the holes, and escape. In section II we will see how such holes can be closed by using time varying circularly polarised rf radiation. In section III we will see that a similar technique with linearly polarised rf will also close such holes, and the paper concludes with a short summary in section IV.

II. A TECHNIQUE TO CLOSE MICROSCOPIC HOLES: THE CIRCULAR POLARISATION CASE

To close the holes in a dressed rf potential we will use a variant of the time-averaged adiabatic potential technique (TAAP) [13]. This technique in its original form involves oscillating the adiabatic potential quite rapidly so that the time averaged potential yields a new potential of interest. In the original example, a ring trap is produced from an egg-shell by a shaking process. In the form used here we will rotate the polarisation direction to vary the adiabatic potential via a time varying Rabi frequency. Then if the mechanical oscillations of the atom are slow enough, we again obtain a time averaged potential so that in the case of holes, the hole is removed by the averaging process. We will also find that we can remove *all orientational* dependence from the potential when the atoms are at a resonant location.
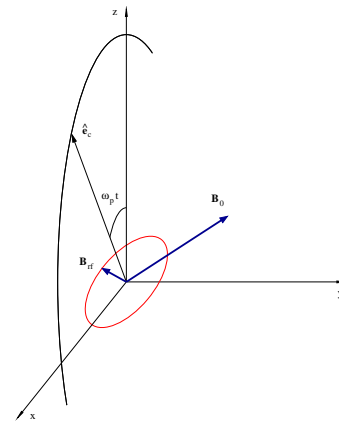

FIG. 1: (Colour online) Schematic showing the co-ordinate system used in section II with a static field $\mathbf{B}_0$ and the rf vector $\mathbf{B}_{\rm rf}$ which rotates rapidly about the polarisation axis $\hat{\mathbf{e}}_c$ with a frequency $\omega_{\rm rf}$. The polarisation axis $\hat{\mathbf{e}}_c$ itself rotates about the y-axis at an angular frequency ω_p.

If we consider first the case of circularly polarised rf, the homogenised coupling can be produced by *rotating the axis of polarisation*. Let us focus on a single location $\mathbf{r}$ and we suppose that the static field $\mathbf{B}_0(\mathbf{r})$ is pointing in an arbitrary direction with components B_{0x}, B_{0y}, and B_{0z}, and that, for concreteness, the circular polarisation axis rotates in the z-x plane with a frequency ω_p (see figure 1). Then for a unit vector in the direction of the polarisation axis we have

$$\hat{\mathbf{e}}_c(t) = \cos \omega_p t \, \hat{\mathbf{z}} + \sin \omega_p t \, \hat{\mathbf{x}} \,. \qquad (6)$$

Within the RWA, the effective interaction is given by equation (5) with the angle θ being the angle between the two vectors $\hat{\mathbf{e}}_c(t)$ and $\mathbf{B}_0(\mathbf{r})$. From the definitions of scalar product and using equation (6)

$$\begin{aligned} \hat{\mathbf{e}}_c(t) \cdot \mathbf{B}_0 &= B_0 \cos \theta \\ &= B_{0x} \sin \omega_p t + B_{0z} \cos \omega_p t \,, \qquad (7) \end{aligned}$$

where $B_0 = \sqrt{B_{0x}^2 + B_{0y}^2 + B_{0z}^2}$. Strictly, the components of $\mathbf{B}_0(\mathbf{r})$ vary with position $\mathbf{r}$, but we omit this in the notation for simplicity. Then, from equation (7),

$$\cos \theta = \frac{B_{0x}}{B_0} \sin \omega_p t + \frac{B_{0z}}{B_0} \cos \omega_p t \,, \qquad (8)$$

which can be inserted into equation (5) to find for the Rabi frequency:

$$\Omega(t) = \Omega_0 \left(1 + \frac{B_{0x}}{B_0} \sin \omega_p t + \frac{B_{0z}}{B_0} \cos \omega_p t \right) . \qquad (9)$$

This assumes a quasi-static situation in which the rotation of polarisation axis is rather slower than the rf frequency. We will now time average the adiabatic potential (3), but we note that in the resonant regime this potential is dominated by the Rabi frequency. As a result

the leading term in the time averaged potential simply involves the time averaged Rabi frequency

$$\overline{\mathcal{E}(\mathbf{r},t)} \sim \hbar F \overline{\Omega(\mathbf{r},t)} = \hbar F \Omega_0(\mathbf{r}) \,. \qquad (10)$$

Strikingly, we see that in the potential (10) all the orientational dependence has been removed, i.e. the direction of $\mathbf{B_0}(\mathbf{r})$ no longer matters. The effective Rabi frequency is $\Omega_0(\mathbf{r})$ which only depends on the magnitude of $\mathbf{B}_{\mathrm{rf}}(\mathbf{r})$ and, in this sense, the vectorial nature of the magnetic field has been eliminated (along with any holes).

The dressed potential (3) shows that trapped atoms will tend to prefer a region of resonance if other forces, such as gravity, can be neglected. If an atom is in the non-resonant regime the detuning $\delta(\mathbf{r}) = \omega_{\mathrm{rf}} - \omega_0(\mathbf{r})$ plays a role in the potential. In the case of a rotating polarisation axis the dressed potential is no longer independent of orientation. To see this we can examine the detuning dominated regime $|\delta(\mathbf{r})| \gg \Omega(\mathbf{r},t)$ where

$$\mathcal{E}(\mathbf{r},t) = \hbar F |\delta(\mathbf{r})| \left[1 + (\Omega(\mathbf{r},t)/\delta(\mathbf{r}))^2 \right]^{1/2} \sim \hbar F |\delta(\mathbf{r})| \left(1 + \frac{\Omega^2(\mathbf{r},t)}{2\delta^2(\mathbf{r})} + ... \right), \qquad (11)$$

so that using equation (9) and performing the time average we find

$$\overline{\mathcal{E}(\mathbf{r},t)} \sim \hbar F |\delta(\mathbf{r})| \left(1 + \frac{\Omega_0^2(\mathbf{r})}{4\delta^2(\mathbf{r})}(3 - B_{0y}^2/B_0^2) + ... \right) \,. \qquad (12)$$

The explicit presence of a term involving B_{0y} demonstrates the orientation dependence in the non-resonant regime.

The validity of the results in this section is ensured by the conditions $\omega_{\mathrm{trap}} \ll \omega_p \ll \omega_0$ [13], where ω_{trap} represents the mechanical oscillation frequency of the atom. The inequality $\omega_{\mathrm{trap}} \ll \omega_p$ ensures that as the atom oscillates in a trap it experiences an average potential $\overline{\mathcal{E}(\mathbf{r},t)}$. At the same time the Larmor frequency must remain a good concept and hence $\omega_p \ll \omega_0$. For a resonant rf atom trap the Larmor frequency ω_0 will be essentially the same as the rf frequency ω_{rf}. The trap frequency can be very low (of order Hz), but the highest values are in a direction transverse to the iso-B surface and depend on the local field gradient and Rabi frequency. For the quadrupole field described in Ref. [7], with 10 MHz rf and a Rabi frequency of 20kHz, the conditions become: 1 kHz $\ll$ 100 kHz $\ll$ 10 MHz. Here a geometric mean of ω_{trap} and ω_{rf} has been taken for the value of the rotation frequency ω_p. For this example the conditions appear to be feasible, but at their limit.

III. ROTATION OF LINEAR POLARISATION DIRECTION

The scheme is less successful in the case of a rotating linear polarisation direction. We can still close any hole, but there remains some orientation dependence. We should note that rotating the linear polarisation axis is in itself not the same as having a circular polarisation. This is because the axis rotation frequency ω_p is supposed to be much less than the rf frequency ω_{rf}. This is a requirement if we want to time average the adiabatic potential, i.e. to make an effective potential for the mechanical motion of the atom whilst the spin dynamics and RWA are both obeyed.

If we take the linear rf polarisation direction to be $\hat{\mathbf{e}}_l(t)$ with, analogously to equation (6),

$$\hat{\mathbf{e}}_l(t) = \cos \omega_p t \, \hat{\mathbf{z}} + \sin \omega_p t \, \hat{\mathbf{x}} \qquad (13)$$

then within the RWA, the effective interaction is given by equation (1). This time, instead of equation (7) we use

$$\begin{aligned} |\hat{\mathbf{e}}_l(t) \times \mathbf{B_0}| &= B_0 |\sin\theta| \\ &= \sqrt{B_{0y}^2 + (\cos\omega_p t B_{0x} - \sin\omega_p t B_{0z})^2} \end{aligned} \qquad (14)$$

Then if we use equation (1) and include the time averaging,

$$\Omega_{\mathrm{eff}} = \overline{\Omega(t)} = \Omega_0 \overline{\sin\theta(t)} \,, \qquad (15)$$

where $\sin\theta$ is to be given by equation (14). The time average is performed using standard integrals to obtain

$$\Omega_{\mathrm{eff}} = \frac{2\Omega_0}{\pi} E(k) \,, \qquad (16)$$

where $E(k)$ is a complete elliptic integral of the second kind with

$$k = \sqrt{1 - \frac{B_{0y}^2}{B_0^2}} \,. \qquad (17)$$

In this case Ω_{eff} does not depend on B_0 alone. In fact Ω_{eff} has values which lie between $2\Omega_0/\pi$ and Ω_0, depending on the relative orientation of the fields.

As in the case of a rotating circular polarisation, an orientational dependence is found in the non-resonant regime. This time we can show that by substituting $\Omega(\mathbf{r},t) = \Omega_0(\mathbf{r})\sin\theta(\mathbf{r},t)$ into the dressed potential (3) and performing the time average we find

$$\overline{\mathcal{E}(\mathbf{r},t)} = 4\hbar F \sqrt{\delta^2(\mathbf{r}) + \Omega_0^2(\mathbf{r})} \quad E(k_\delta) \qquad (18)$$

where

$$k_\delta(\mathbf{r}) = \frac{\Omega_0(\mathbf{r})}{\sqrt{\delta^2(\mathbf{r}) + \Omega_0^2(\mathbf{r})}} \quad k(\mathbf{r}) \,. \qquad (19)$$

The source of the orientational dependence is $k(\mathbf{r})$, which is given by equation (17).

IV. SUMMARY

The origin of holes in dressed state potentials is the *vectorial* nature of the magnetic field interaction itself.

We should emphasise that not all dressed state systems produce holes. In the case of linearly polarised rf, the strongest coupling (Rabi frequency) is obtained when the rf polarisation is orthogonal to the static magnetic field. Within the rotating wave approximation (RWA) there is no coupling at all if the polarisation is in the same direction as the static magnetic field $\mathbf{B_0}$: this is essentially the source of a hole.

A hole may persist even if circular polarisation is used. However, we have seen that by continuously rotating the polarisation axis, the hole can be eliminated in a time averaged adiabatic potential (TAAP). Furthermore the coupling becomes independent of the relative orientation of the static and rotating magnetic fields (equation (10)): in effect the vector nature of the magnetic field is lost. The hole is removed even if a rotating linear polarisation is used. However, in that case, the resulting coupling is not completely independent of orientation. For off-resonant atoms the dressed potentials are never independent of field orientation for the schemes considered here. So although there are situations where complete dressed rf potential surfaces have no holes, even if there are holes, those holes can be circumvented by the use of the TAAP technique. This adds to the flexibility and applicability of trapping with dressed rf potentials.

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